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Example of Linearly Dependent and Linearly Independent Vectors

1. Examine if the set of vectors $\{(2,1,1),(1,2,2),(1,1,1)\}$ is linearly dependent in $\mathbb{R}^3$.

Solution :→ Let, $\alpha=(2,1,1)$, $\beta=(1,2,2)$ and $\gamma=(1,1,1)$.

Let us consider the relation $c_1\alpha+c_2\beta+c_3\gamma=\theta$, where $c_1, c_2, c_3 \in \mathbb{R}$ and θ is the zero element of $\mathbb{R}^3$.

Then $c_1(2,1,1)+c_2(1,2,2)+c_3(1,1,1)=(0,0,0)$

$\Rightarrow (2c_1+c_2+c_3,\ c_1+2c_2+c_3,\ c_1+2c_2+c_3)=(0,0,0)$.

$\therefore\ 2c_1+c_2+c_3=0,\ c_1+2c_2+c_3=0,\ c_1+2c_2+c_3=0$.

i.e. $2c_1+c_2+c_3=0$, $c_1+2c_2+c_3=0$ $\Rightarrow \dfrac{c_1}{1-2}=\dfrac{c_2}{1-2}=\dfrac{c_3}{4-1}=k$, say

$\Rightarrow c_1=-k,\ c_2=-k,\ c_3=3k$.

, where k is a real number.

Since k is arbitrary, so $\exists$ c_1, c_2, c_3, not all zero, s.t. $c_1\alpha+c_2\beta+c_3\gamma=\theta$. [For example $c_1=-1$, $c_2=-1$, $c_3=3$]

$\therefore$ The set of vectors is linearly dependent.

2. Prove that the set of vectors $\{(1,2,2),(2,1,2),(2,2,1)\}$ is linearly independent in $\mathbb{R}^3$.

Solution :→ Let, $\alpha=(1,2,2)$, $\beta=(2,1,2)$, and $\gamma=(2,2,1)$.

Let us consider the relation $c_1\alpha+c_2\beta+c_3\gamma=\theta$, where $c_1, c_2, c_3 \in \mathbb{R}$ and θ is the zero element of $\mathbb{R}^3$

$\therefore\ c_1(1,2,2)+c_2(2,1,2)+c_3(2,2,1)=(0,0,0)$

$\Rightarrow c_1+2c_2+2c_3=0,\ 2c_1+c_2+2c_3=0,\ 2c_1+2c_2+c_3=0$

This is a homogeneous system of equations in c_1, c_2 and c_3.

The coefficient determinant of the system is $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$=1(1-4)+2(4-2)+2(4-2)$

$=-3+4+4=5\neq 0$

By Cramer's rule, $\exists$ a unique solution for c_1, c_2 and c_3 and the solution is the trivial solution $c_1=0$, $c_2=0$ and $c_3=0$

$\therefore$ The given set of vectors is linearly independent.